

Enhancement for Reduced Basis Methods

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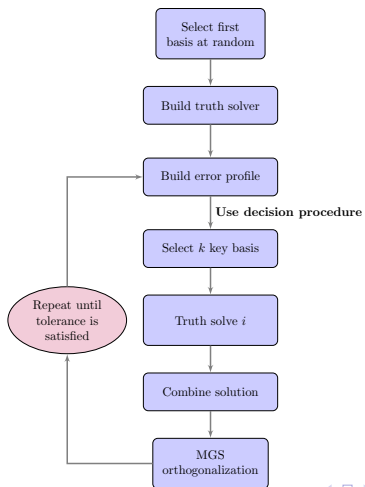
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Reduced Basis Methods

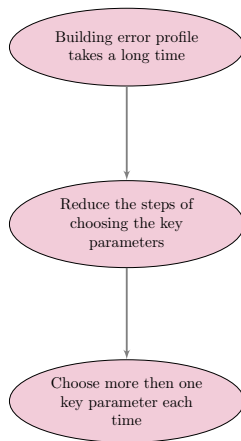
- Offline: Greedy Algorithm



- Online: Use the parameter basis we chose to represent the solution

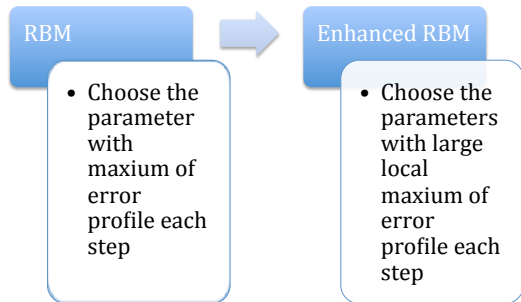
$$\left. \begin{aligned} u(x, \mu_*) &= \sum_{i=1}^n c_i u(x, \mu_i) \\ Lu(x, \mu_*) &= f(x, \mu_*) \end{aligned} \right\} \Rightarrow \{c_i\}_{i=1}^n$$

Motivation



- Reduce the computational costs of the offline procedure
- Guarantee the quality of the key parameter basis

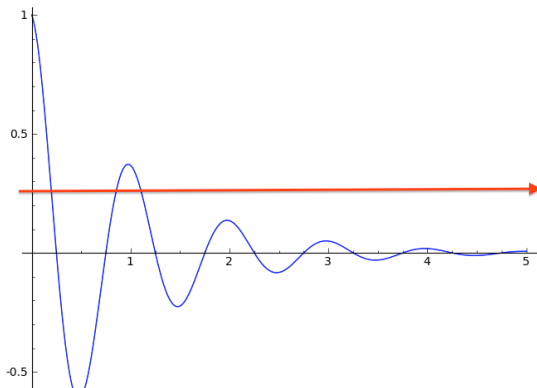
Enhancement for RBM



Enhancement for RBM

Way of choosing the parameters with local maximum error profile

- Sort the error profile: $E(\mu_{j1}), E(\mu_{j2}), E(\mu_{j3}), \dots, E(\mu_{jn})$ from high to low
- Choose them one by one from high to low until $\sum_{s=1}^{s=k} E(\mu_{js}) / \text{sum} \geq 60\%$, $\text{sum} = \sum_{i=1}^{i=n} E(\mu_{ji})$



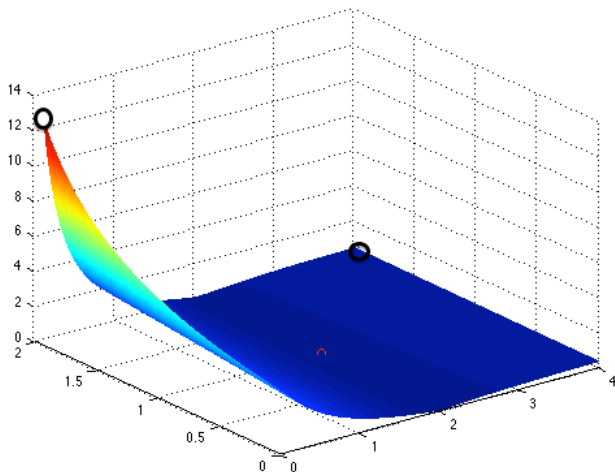
We use a Anisotropic wave speed Equation as example:

$$-u_{xx} - \mu_1 u_{yy} - \mu_2 u = -10 \sin(8x(y - 1))$$

on $[-1, 1] \times [-1, 1]$ with $u = 0$ on the boundary

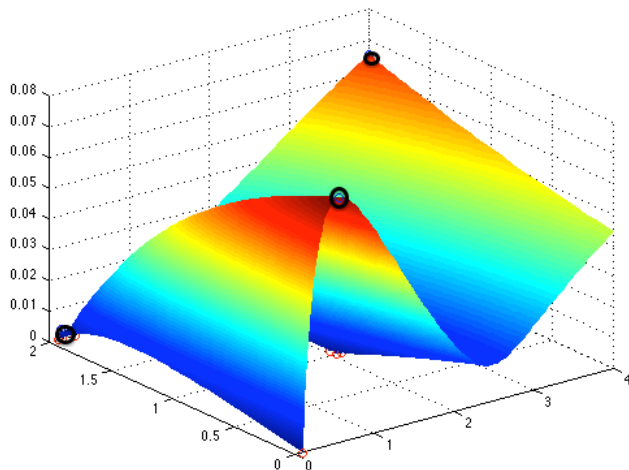
Choose key parameters

- First step



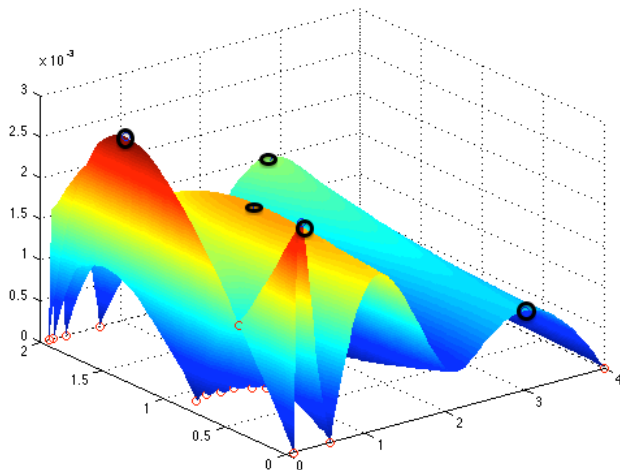
Choose key parameters

- Fourth step



Choose key parameters

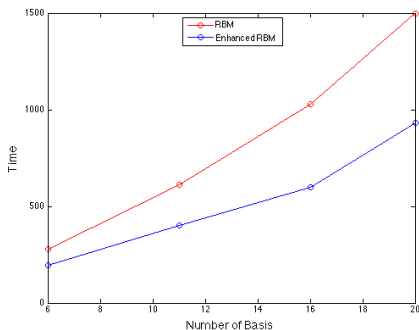
- Six step



Numerical Result

- Comparison of Time

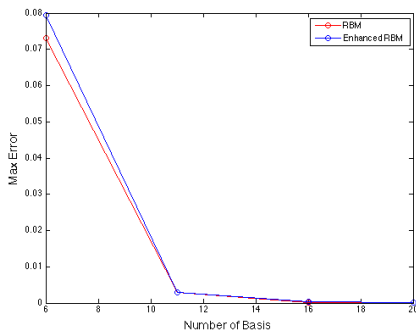
Number of Basis	RBM(s)	Enhanced RBM(s)
6	277.874	195.470
11	611.234	399.535
16	1029.341	600.415
20	1498.350	930.712



Numerical Result

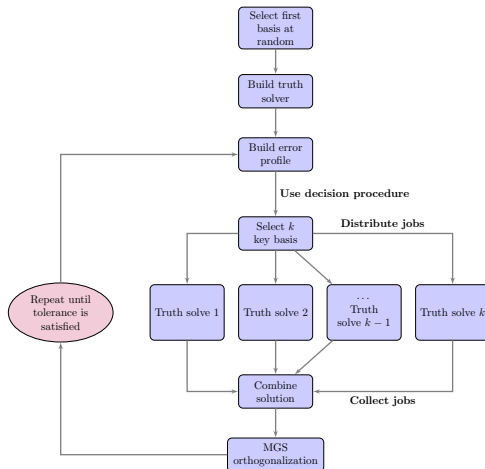
- Comparison of Maximum Error

Number of Basis	RBM	Enhanced RBM
6	0.0731	0.0795
11	0.0030	0.0029
16	0.00021025	0.00028719
20	0.000022180	0.000022174



Future

- Parallelize the part of building error profile and true solver



Question ?